JOURNAL OF MATERIALS SCIENCE 28 (1993) 2687–2692

Modelling the effects of negative Poisson's ratios in continuous-fibre composites

M. A. NKANSAH, K. E. EVANS
Department of Materials Science and Engineering, The University of Liverpool, PO Box 147, Liverpool L69 3BX, UK
I. J. HUTCHINSON,
Research and Technology Department, ICI Chemicals and Polymers Ltd, PO Box 8, The Heath, Runcorn, Cheshire WA7 4QE, UK

Finite-element methods have been used to examine the elastic properties of continuous-fibre reinforced composites. Consideration has been given to the possibilities of using either reinforcing or matrix constituents with large negative Poisson's ratios (with values of $-1.0 \leqslant \nu \leqslant -0.3$). It is shown that a large negative Poisson's ratio can lead directly to considerably enhanced transverse moduli without altering the longitudinal moduli. For example, changing the matrix Poisson's ratio from 0.3 to -0.9 and keeping all other constituent properties constant produces an almost four-fold increase in the composite transverse modulus.

1. Introduction

Naturally occurring materials exhibit a positive Poisson's ratio, which means that under a tensile load there is a lateral contraction. There is now increasing interest in a new class of materials, termed auxetics or auxetic materials, which expand laterally when stretched and contract laterally when compressed. Several examples now exist of materials demonstrating this effect [1–6]. Mechanisms have been identified that may operate within the material either macroscopically [7–9], microstructurally [10, 11] or at the molecular level [12] to produce the effect. In all cases the effect is produced by non-central forces acting to give a bidirectional displacement under the action of a unidirectional load. One of the most common examples quoted is the re-entrant dovetail shape [8, 12, 13] which was first used as a model for the behaviour of auxetic foams. In this case deformation is by flexure of the cell walls. More recently tensile network structures have been identified that produce similar effects. In all cases the material undergoes an increase in volume when stretched.

The advantages of a negative Poisson's ratio have been discussed [1, 14] and include improved shear moduli, indentation resistance and plane strain fracture toughness. Auxetic materials that have been produced include honeycombs [13], polymer foams [1], metal foams [2], microporous polymers [3, 5] and polymer gels [6]. Recently it has also been shown that an auxetic material can be designed at the molecular level [12] and synthesis routes for such materials have been proposed [15]. This opens up the possibility of producing materials with the same range of interesting properties but that are intrinsically much stronger and stiffer.

Composite materials exhibiting a negative Poisson's ratio have been examined theoretically [9, 16, 17] and experimental evidence for benefits in such systems is accumulating [18, 19]. A negative Poisson's ratio in a composite may be achieved in a number of ways. Firstly, suitable network structures can be created in a composite made of conventional materials [17] (so-called "network-embedded" composites), for example a re-entrant network of high-modulus material embedded in a low-modulus matrix. Alternatively either the matrix or the reinforcement itself may be auxetic. These two latter cases are the subject of this paper. Fibrous structures can be made from auxetic microstructures, as has been seen in polymeric materials [3]. Alternatively either fibres or matrix may be fabricated from molecular auxetics.

In the case of network-embedded composites, simple rule-of-mixtures approaches to approximate the elastic properties have not only been shown to be quantitatively very inaccurate but they also fail to predict the existence of novel effects [17]. Finite-element methods were used to model the microstructures. In this paper we return to the conventional continuous-fibre composite with either matrix or fibre materials having negative Poisson's ratios. Again a rule of mixtures approach is not expected to apply since assumptions of uniform Poisson's ratios are made in such models.

The elastic properties of conventional unidirectional fibre-reinforced composites can reasonably be predicted by conventional rule-of-mixtures expressions, provided the usual assumptions are made. An earlier paper [17] has shown that although the results of rule-of-mixtures calculations are borne out by results obtained from finite-element analysis for longitudinal loading of the undirectional composite, there is no agreement for transverse loading. The purpose of this paper is to study how well the rule-of-mixtures predictions of elastic properties agree with that of finite-element analysis for a continuous-fibre composite made from fibres and/or matrix materials with negative Poisson's ratio, and to see the possible beneficial effects produced by using auxetic materials.

2. Finite-element model

A two-dimensional representation of a unidirectional fibre reinforced composite is shown in Fig. 1 with the matrix material shaded. The longitudinal axis is shown by 1 and the transverse axis by 2. For convenience a two-dimensional model is used for direct comparison with the analytical model. By the application of appropriate boundary conditions the representation in Fig. 1 can be taken as a unit cell whose replication throughout space defines the material. Tensile loading is applied both longitudinally and transversely to determine the composite Young's moduli E_1, E_2 and Poisson's ratios ν_{12}, ν_{21}.

It is only necessary to model one-quarter of the cell and apply appropriate boundary conditions because of the symmetry of the reinforcement and loading applied. Such models are shown for two cases of differing fibre volume fraction in Fig. 2a and b. The two lines of mirror symmetry on the 1 and 2 axes have roller-bearing boundary conditions to allow for transverse contraction or expansion under tensile loading. Loading is applied by a small displacement perpendicular to one of the two free edges, the choice depending on whether we wish to apply longitudinal or transverse loading. Typically, values of strain of about 5% were used. The finite-element grids shown in Fig. 2a and b comprised eight noded quadrilaterals and a proprietary finite-element package [20] was used.

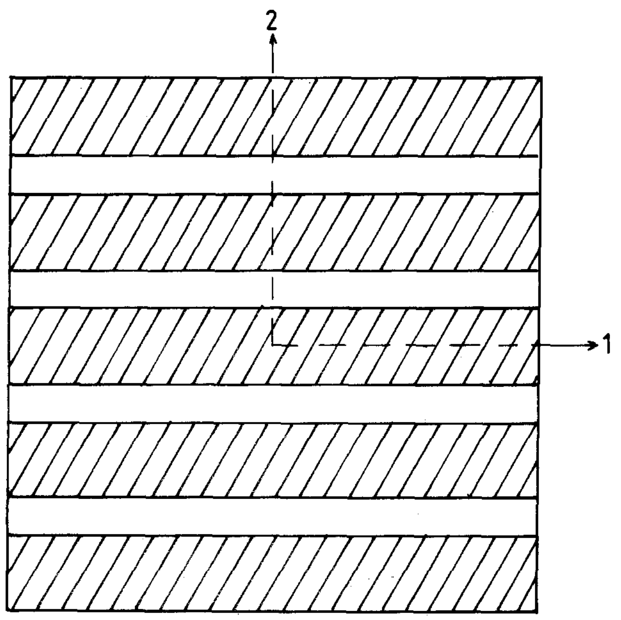

Figure 1 Unit cell of continuous-fibre composite. Fibre reinforcement is unshaded, matrix is shaded region.

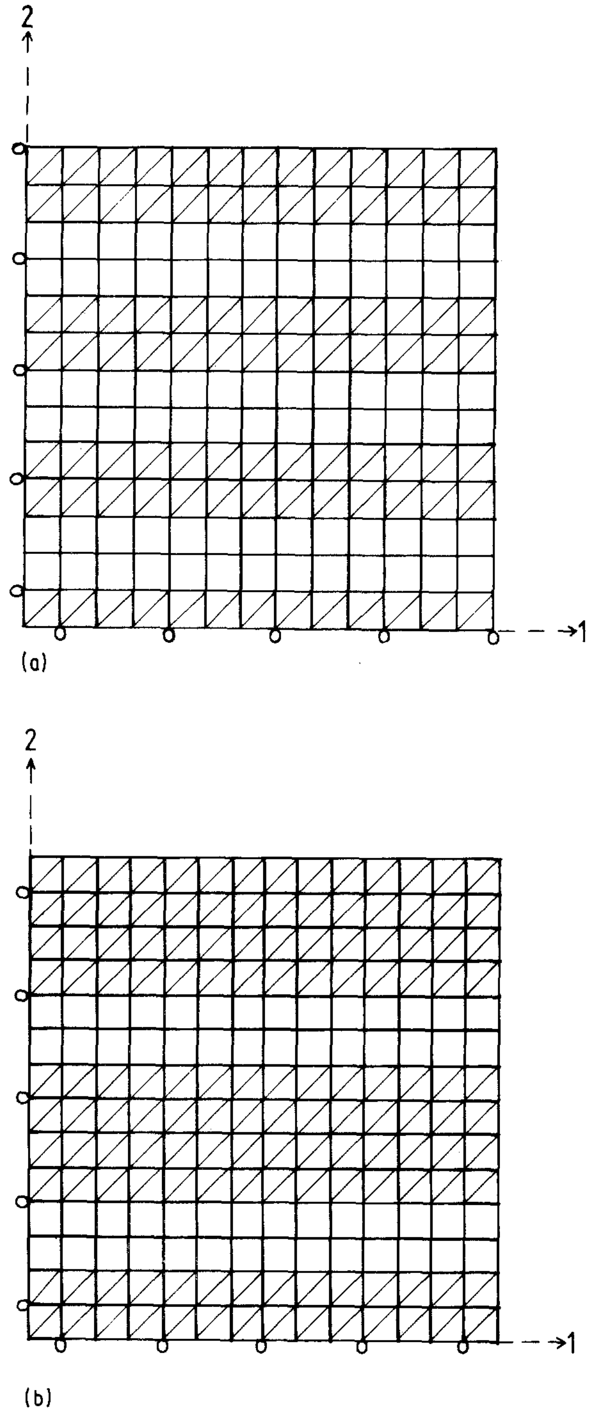

Figure 2 Finite element grids (a) for case 1 ($V_f = 0.462$), (b) for case 2 ($V_f = 0.286$).

A typical deformation of the model under a longitudinal tensile load is shown in Fig. 3 for a composite with conventional constituents. The matrix is left unshaded to make the deformation clearer. There is a constant transverse displacement on the transverse edge and so it is a straightforward matter to determine the Poisson's ratio. However, under transverse loading the fibres and matrix undergo different displacements if left to displace freely. This condition cannot occur in the real material since the transverse boundaries of any repeating unit cell within the bulk material will remain straight due to symmetry.

This problem has been overcome [17] by applying a moving constraint boundary condition at the free edge as described in detail elsewhere [17]. The method

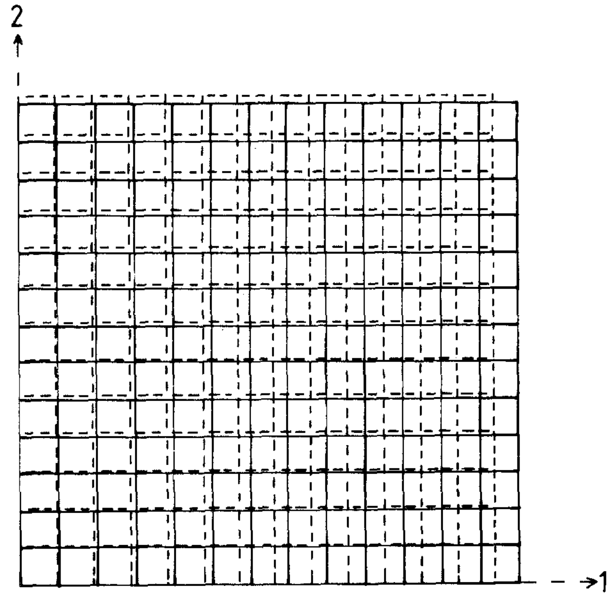

Figure 3 Typical deformed grid (solid lines, undeformed grid shown in broken lines) under longitudinal loading (along 1 axis).

consists of coupling together all the appropriate degrees of freedom (in this case they are the 1-directed freedoms on the face perpendicular to the 1-direction) so that they have the same displacement. This means that one of the unknown variables in each of the simultaneous equations to be solved, to obtain nodal displacements, is chosen to have some unknown value. This approach is explained in standard texts [21] on the finite-element method.

3. Verification of model accuracy

For continuous-fibre composites with loading parallel to the fibres, experimental and theoretical work [22–24] have shown that the simple rule-of-mixtures equations for the longitudinal composite modulus (E_1) and the major Poisson's ratio (ν_{12}) are accurate within 1 or 2% for Poisson's ratios in the range 0.2–0.4. These are

$$E_1 = E_f V_f + E_m V_m \tag{1}$$

and

$$\nu_{12} = \nu_m V_m + \nu_f V_f \tag{2}$$

where E_f, ν_f, V_f and E_m, ν_m, V_m are the Young's modulus, Poisson's ratio and volume fraction for the fibre and matrix, respectively.

The accuracy of the finite-element model can therefore be tested by comparison of the results from it with that obtained from the above two equations. These equations have been tested and accurately represent the Young's modulus and Poisson's ratio under various combinations of E_f and E_m and further details are given in an earlier paper [17].

4. Results

Table I summarizes the material properties and volume fractions used in the finite-element models. The rule-of-mixtures equations for longitudinal loading of the continuous-fibre composite were given in the previous section. For transverse loading the transverse composite modulus (E_2) is approximately given by

$$E_2 = \left(\frac{V_f}{E_f} + \frac{V_m}{E_m}\right)^{-1} \tag{3}$$

and

$$\nu_{21} = \nu_{12}\frac{E_2}{E_1} \tag{4}$$

Equation 3 has been shown experimentally to give

TABLE I Geometries and properties for continuous-fibre composite models

Geometries	
Case 1: fibre volume fraction $V_f = 0.462$	
Case 2: fibre folume fraction $V_f = 0.286$	
Property variations	
Moduli: $E_f = 76$ GPa, $E_m = 3$ GPa	
Poisson's ratio ν_f at	-0.9, -0.7, -0.5, -0.3, 0.3, 0.49
Poisson's ratio ν_m at	-0.9, -0.7, -0.5, -0.3, 0.3, 0.49

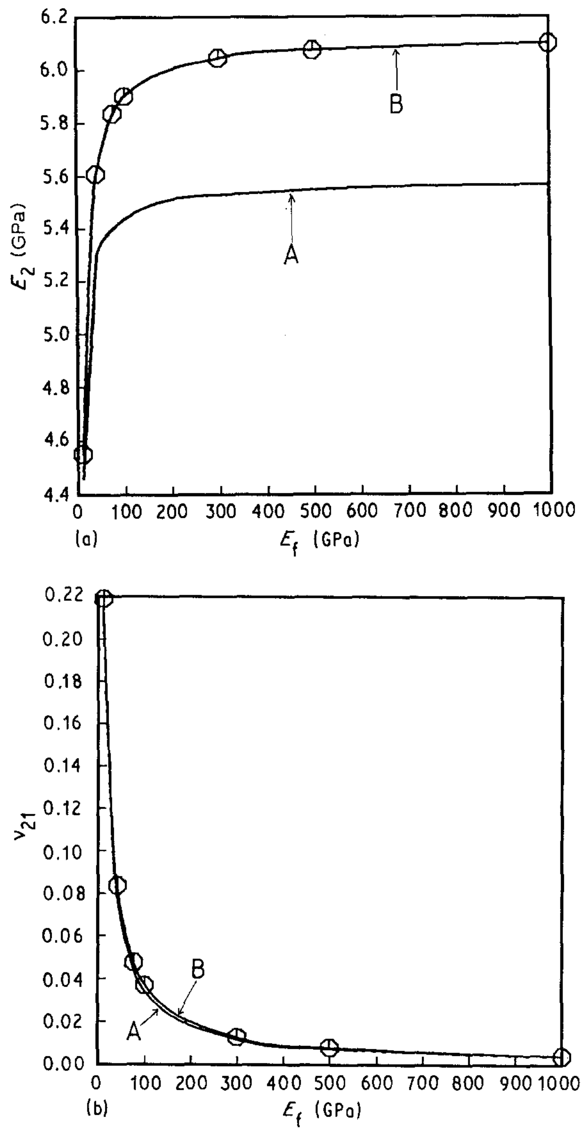

Figure 4 Plots of (a) transverse composite modulus E_2 and (b) composite Poisson's ratio ν_{21} versus fibre modulus E_f. (A) Case 1, rule of mixtures; (B) case 1, finite-element analysis, $\nu_f = \nu_m = 0.3$.

rather poor results [22, 23]. This is borne out in Fig. 4a which is a plot of E_2 against fibre modulus when $\nu_f = \nu_m = 0.3$ and $E_m = 3$ GPa. Surprisingly, the agreement between the rule-of-mixtures results and the finite-element results for ν_{21} is much closer as can be seen in Fig. 4b.

In Fig. 5a ν_{12} is plotted as a function of the fibre Poisson's ratio with the matrix Poisson's ratio fixed at 0.3 while in Fig. 5b it is plotted as a function of the matrix Poisson's ratio with that of the fibre fixed at 0.3. In both cases $E_f = 76$ GPa and $E_m = 3$ GPa. Both figures show the exact agreement between the rule-of-mixtures and finite-element results. There was also exact agreement between the rule-of-mixtures and finite-element results for E_1 as the Poisson's ratios of the fibre and matrix constituents were independently varied. This shows that the accuracy of E_1 and ν_{12} are unaffected by the differences in the Poisson's ratios of the constituents in a continuous fibre composite.

The effect of applying transverse loading can be seen for E_2 in Fig. 6a and b and for ν_{21} in Fig. 7a and b when either (a) the fibre or (b) the matrix Poisson's ratio is varied. The discrepancy between the rule-of-mixtures and finite-element results for E_2 as the fibre Poisson's ratio is varied is clearly seen in Fig. 6a. In Fig. 6b where the matrix Poisson's ratio is varied we see a most interesting effect. For $\nu_m = 0$ we obtain exact agreement between Equation 3 and the finite-element results. However as ν_m varies away from zero the effective modulus increases. Considering the matrix Poisson's ratio (ν_m) between -0.5 and 0.5 there is clearly symmetry about $\nu_m = 0$. It is clear that not only is the rule-of-mixtures formula unable to predict the non-linear nature of the variation between E_2 and ν_m, but it also fails to show the rapidly increasing value of E_2 as a result of the interaction between the constituents of the composite due to the difference in their Poisson's ratios. In particular, for $\nu < -0.5$ a very considerable increase in E_2 occurs.

In Fig. 7a and b ν_{21} is plotted against ν_f and ν_m, respectively. Again it can be seen that there is a very large variation of ν_{21} for $\nu_m < -0.5$.

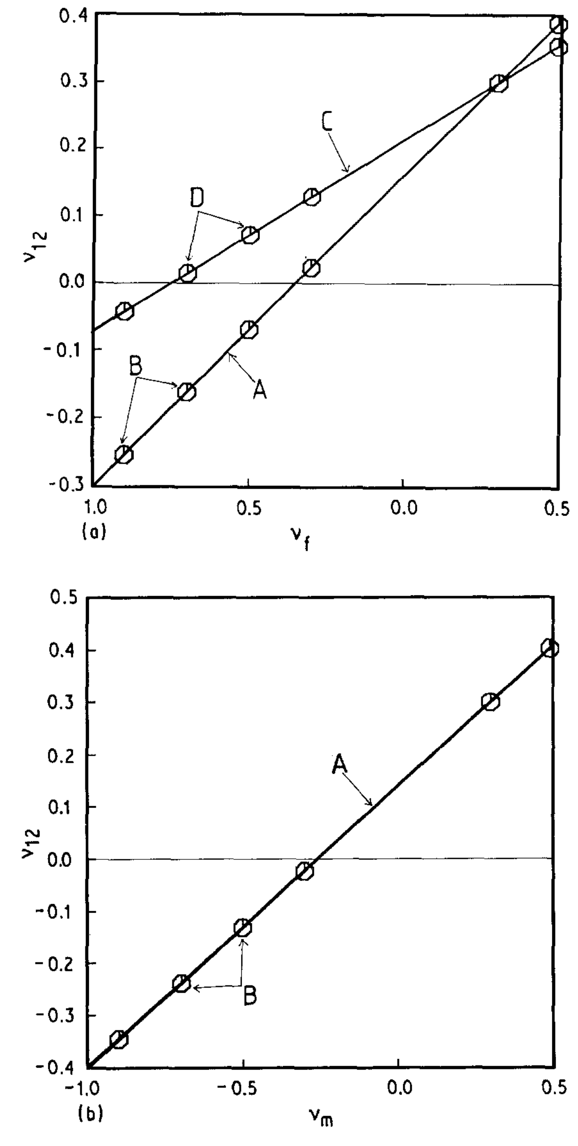

Figure 5 Plot of longitudinal composite Poisson's ratio ν_{12} versus (a) fibre Poisson's ratio and (b) matrix Poisson's ratio (A) Case 1, rule of mixtures; (B) case 1, finite-element analysis; (C) case 2, rule of mixtures; (D) case 2, finite-element analysis.

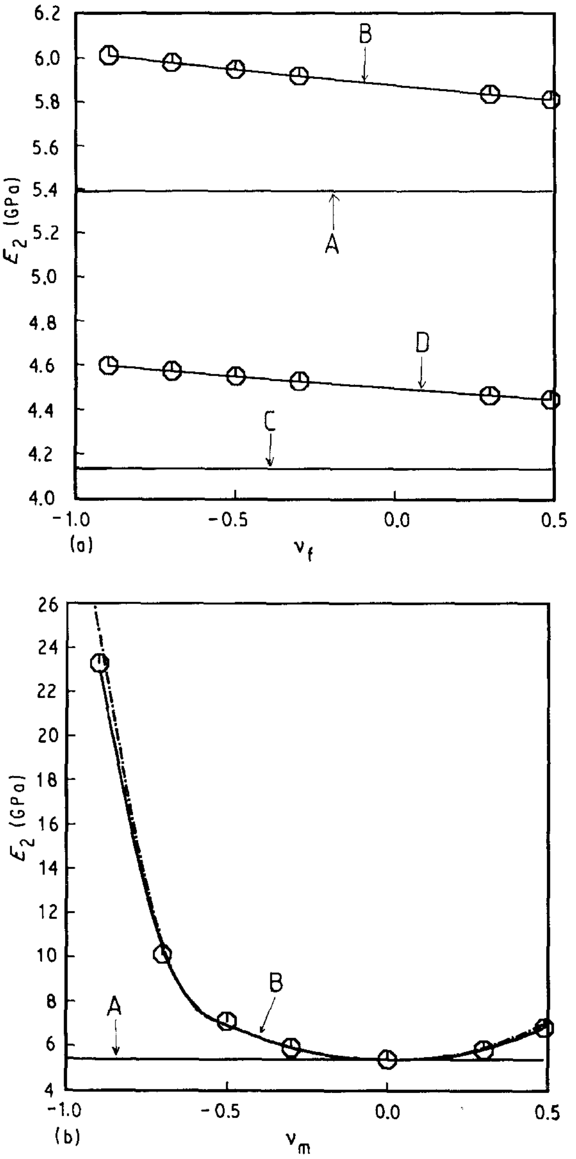

Figure 6 Plot of transverse composite modulus E_2 versus (a) fibre Poisson's ratio and (b) matrix Poisson's ratio. (A) Case 1, rule of mixtures; (B) case 1, finite-element analysis; (C) case 2, Rule of mixtures; (D) case 2, finite-element analysis. The dashed line in Fig. 6b represents the modified Equation 3 with E_m replaced by $E_m\,(1-\nu_m^2)^{-1}$.

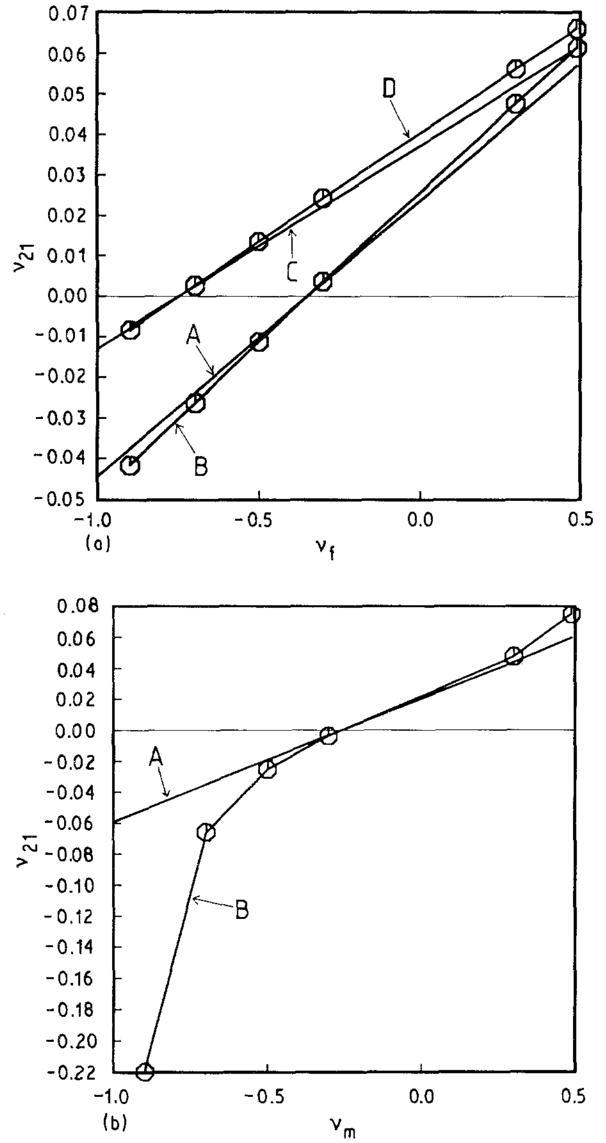

Figure 7 Plot of transverse composite Poisson's ratio v_{21} versus (a) fibre Poisson's ratio and (b) matrix Poisson's ratio. (A) Case 1, rule of mixtures; (B) case 1, finite-element analysis; (C) case 2, rule of mixtures; (D) case 2, finite-element analysis.

5. Discussion

The results from the rule-of-mixtures Equations 1 and 2 for E_1 and v_{12} for a continuous-fibre composite agree with the finite-element results for all combinations of properties of the constituent fibre and matrix considered in this paper. Although Equations 1 and 2 are not strictly exact expressions they have been shown to be accurate within experimental error [22–24]. Hence E_1 is unaffected by changes in either the matrix or fibre Poisson's ratios.

The relationship in Equation 3 and consequently in Equation 4 has, however, been found not to hold even where the Poisson's ratio of the fibre and matrix are both equal and positive. Early investigators [22, 23] have shown that the value of E_2 given by Equation 3 constitutes a lower bound on the modulus of elasticity of macroscopically isotropic composites, and so represents a lower bound on composite Young's modulus irrespective of phase geometry.

The most interesting result obtained here is the extent of the improvement to the composite transverse modulus. E_2, for matrix Poisson's ratios less than -0.3 (see Fig. 6b). The simple rule-of-mixtures expression of Equation 3 assumes equal Poisson's ratios in both constituents and does not allow for differential contractions in the matrix and reinforcement. As the composite is loaded the matrix tries to deform laterally. This lateral deformation is resisted by the much stiffer reinforcement. The resulting reduction in lateral displacement produces a concommitant reduction in the loading direction, the extent of which is defined by the matrix's Poisson's ratio. For the case of an infinitely stiff reinforcement the matrix is effectively in a state of plane strain and the apparent modulus of the matrix is increased by a factor $(1\text{-}v_m^2)^{-1}$. In this case Equation 3 is modified by $E_m \rightarrow E_m'$ where $E_m' = E_m(1 - v^2)^{-1}$.

The result of this approximation improves the prediction of E_2 [24]. However for $v_m \simeq 0.3$ the change is less than 10%. For isotropic materials the full range of values available is $-1 \leqslant v \leqslant \frac{1}{2}$. So for v approaching -1 the multiplicative factor approaches infinity. This assumes, of course, that the fibre is infinitely stiff. This modified relationship is plotted in Fig. 6b and represents an upper limit of improvement to E_2 simply by altering the value of v_m alone. It is quite surprising how close the results are. The value of $E_f = 76$ GPa represents a very conservative value of the reinforcing modulus (e.g. for a glass fibre). By using carbon or Kevlar fibres ($E_f > 500$ GPa possible) we would expect an even closer fit to the theoretical prediction. Altering v_f makes a relatively small change to E_2 (Fig. 6a) because the matrix modulus is too low to significantly affect the fibre lateral strain.

This explanation can also be applied to the longitudinal case. Here there is no restriction on the Poisson contraction, because of the axial symmetry of the reinforcement, and hence E_1 is not altered by changing v_f or v_m.

For a fibre volume fraction of $V_f = 0.462$ and $v_f = v_m = 0.3$ we obtained $E_1 = 37.0$ GPa, $E_2 = 6.0$ GPa, $v_{12} = 0.3$, $v_{21} = 0.05$ [17]. When the matrix Poisson's ratio was altered to $v_m = -0.9$ but all other properties were kept constant we obtained $E_1 = 37.0$ GPa, $E_2 = 23.0$ GPa, $v_{12} = -0.34$ and $v_{21} = -0.22$. Thus the composite is much more nearly isotropic and E_2 has increased nearly four times. The theoretical maximum improvement for $v_m = -0.9$ would be 4.22 times the rule of mixtures value.

6. Conclusions

It has been shown that if the constituents of a continuous-fibre composite consist of auxetic materials the conventional rule-of-mixtures equations can be used to accurately predict the longitudinal composite modulus and Poisson's ratio. The simple rule-of-mixtures expressions provides a lower bound for the transverse modulus. By modifying this to allow for Poisson contraction of the matrix an upper bound is obtained that is reasonably accurate for most continuous-fibre composites. Most importantly, an auxetic material can be used as the matrix in a continuous-fibre composite in order to sharply increase the value of the transverse composite modulus without reduction of the longitudinal modulus.

Acknowledgements

M. A. N. and K. E. E. would like to acknowledge the financial support of ICI C & P Ltd, and K. E. E. would like to acknowledge the support of an SERC Advanced Fellowship during this work.

References

1. R. S. LAKES, *Science* **235** (1987) 1038.
2. E. A. FRIIS, R. S. LAKES and J. B. PARK, *J. Mater. Sci.* **23** (1988) 4406.
3. B. D. CADDOCK and K. E. EVANS, *J. Phys. D. Appl. Phys.* **22** (1989) 1877.
4. K. E. EVANS and B. D. CADDOCK, *ibid.* **22** (1989) 1883.
5. K. L. ALDERSON and K. E. EVANS, *Polymer* **33** (1992) 4435.
6. S. HIROTSU, *J. Chem. Phys.* **94** (1991) 3949.
7. L. J. GIBSON, M. F. ASHBY, G. S. SCHAJER and C. I. ROBERTSON, *Proc. R. Soc. London* **A382** (1982) 25.
8. R. F. ALMGREN, *J. Elast.* **15** (1985) 427.
9. C. T. HERAKOVICH, *J. Compos. Mater.* **18** (1985) 447.
10. K. E. EVANS, *J. Phys. D. Appl. Phys.* **22** (1989) 1870.
11. R. S. LAKES, *J. Mater. Sci.* **26** (1991) 2287.
12. K. E. EVANS, M. A. NKANSAH, I. J. HUTCHINSON and S. C. ROGERS, *Nature* **353** (1991) 124.
13. L. J. GIBSON, and M. F. ASHBY, "Cellular Solids; Structure and Properties", (Pergamon, Oxford, 1988).
14. K. E. EVANS, *Chem. Ind.* **20** (1990) 654.
15. G. WEI and S. F. EDWARDS (Cavendish Laboratory, Cambridge University) Private Communication (1992).
16. G. W. MILTON, "Composite Materials with Poisson's Ratios Close to -1", *J. Mech. Phys. Solids* **40** (1992) 1105.
17. K. E. EVANS, M. A. NKANSAH and I. J. HUTCHINSON, *Acta Met. Mater.* **40** (1992) 2463.
18. J. CHAUDHURI and Q. JANG, in Proceedings of American Society for Composites 3rd Technical Conference, (Technomic, Lancaster, USA, 1988) p. 701.
19. J. P. DONOGHUE and K. E. EVANS, in Proceedings of 8th International Conference on Composite Materials, edited by (S. W. Tsai and G. S. Springer) (SAMPE Publications, Covina, 1991).
20. ANSYS, (Swanson Analysis Systems Inc., P O Box 65, Johnson Road, Houston, USA).
21. I. M. SMITH, "Programming the Finite Element Method", (Wiley, New York, 1982) p. 48.
22. R. HILL, *J. Mech. Phys. Solids* **12** (1964) 199.
23. S. W. TSAI, J. C. HALPIN and N. J. PAGANO, "Composite Materials Workshop", (Technomic, Stamford, Connecticut, USA, 1968) p. 130.
24. D. HULL, 'An Introduction to Composite Materials' (Cambridge University Press, Cambridge, 1981) p. 83.

Received 30 September 1991
and accepted 11 August 1992